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EFFECT OF TEMPERATURE ON NATURAL FREQUENCY OF LAMINATED COMPOSITE PLATE

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ABSTRACT

Laminated composite plates are very important engineering structure with widely application in aerospace, automobile, and nuclear industry due to their high specific strength and specific modulus. In recent years, these materials have been used extensively in advanced aircraft. In such cases certain areas of the aircraft structure may be subjected to thermal loading. It is therefore necessary to predict the free vibration response of the composite plate when subjected to thermal loadings. Several investigators have analyzed the free vibration of the laminated composite plates but the effect of temperature on the free vibration of Carbon Fibre Reinforce Polymer (CFRP) plates has not been extensively investigated. First-order shear deformation theory was applied in Finite Element Method (FEM) to free vibration analysis of fully clamped laminated plate. A theoretical equation was developed and compared it with FEM result. The layup investigated were $[0_2\ ,\ 90_2]$, $[0\ ,90]_4$, $[\pm 45_2]$, $[\pm 45]_4$, $[0,\ 90]_{2s}$, $[\pm 45]_{2s}$, $[0,\ \pm 45,\ 90]_s$ The parametric study showed that the single cross ply $[0_2, 90_2]$ and double angle ply $[\pm 45_2]$ laminates showed greatest variations in natural frequencies. Only slight variations were predicted for the multi-ply laminates $[0, 90]_{4,}$ $[\pm 45]_4$, $[0, 90]_{2s}$, $[\pm 45]_{2s}$, [0, ±45, 90]s.

Keywords: Natural Frequency, CFRP, FSDT, FEM, Laminated Composite Plate.

1. INTRODUCTION

Laminated composite plates are very important engineering structure with widely application in aerospace, automobile, and nuclear industry due to their high specific strength and specific modulus. Furthermore, material properties of laminated composite plates can be tailored to optimize the desired characteristics. For example, materials need to maximize the fundamental frequency and to minimize the maximum deflection for given loads and boundary conditions. The free vibration behavior of laminated composite plates is considerably more complicated than that of isotropic ones due to the anisotropic of the individual lamina and unsymmetric layering. Composite plate of Carbon fibre reinforced polymer (CFRP) is now used extensively in modern aircraft structures. In recent years, these materials have been used extensively in advanced V/STOL aircraft such as McAIr AV-8B/GR MK5 [1]. In such cases certain areas of the aircraft structure may be subjected to thermal loading. It is therefore necessary to predict the free vibration response of the composite plate when subjected to thermal loadings. To reveal the thermal effect on the vibration of a plate with uniform temperature change, it is either that the mechanical properties of the plate are considered as functions of temperature [2-3] or, the nonlinear plate theories, or the nonlinear strains should be applied [4-5]. The dynamic response of simply supported and clamped CFRP composite plate subjected to thermal environment is carried out using the general purpose finite element program ANSYS.

2. COMPOSITE LAMINATED PLATE THEORIES

There are several theories currently in use for the composite laminated plates [Reddy (1997)]. The simplest one the classical laminated plate theory (CLPT). Composites have a very low transverse shear modulus compared to their in-plane moduli. Therefore, the CLPT may not be sufficient for the dynamic analysis of composite plates. To account for the shear deformation, the first order shear deformation theory (FSDT) can be adopted for the analysis. The FSDT assumes that the transverse normals are straight but not perpendicular to the mid surface after deformation. Thus transverse shear strains are constant through the cross section. Since the actual shear stress is not constant, FSDT uses a shear correction factor. In FEM analysis FSDT is used.

For formulating theoretical equation consider a rectangular plate with dimensions L_x and L_y simply supported along its four edge. The layup of the plate is symmetrical, [B] = 0. The mass of the plate is uniform

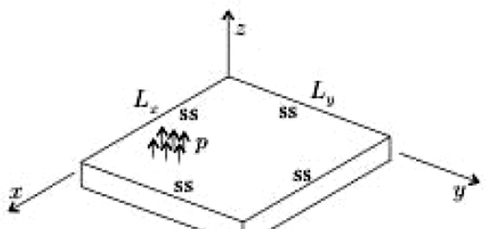

Fig 1. Rectangular Simply Supported Plate

For a linear elastic system the strain energy of volume V is defined as

$$U = \frac{1}{2}\iiint\left(\varepsilon_x\ \sigma_x + \varepsilon_y\sigma_y + \varepsilon_z\ \sigma_z + \gamma_{yz}\tau_{yz} + \gamma_{xz}\tau_{xz} + \gamma_{xy}\tau_{xy}\right)dV \qquad (1)$$

For plane stress assumption strain energy simplifies to

$$U = \frac{1}{2}\int_0^{L_x}\int_0^{L_y}\int_{h_b}^{h_t}\left(\ \varepsilon_x\ \sigma_x + \varepsilon_y\sigma_y + \gamma_{xy}\tau_{xy}\ \right)dzdydx \qquad (2)$$

For a simply supported plate (symmetrical layup) subjected to out-of-plane loads only, the in plane strains in the midplane are zero.

$$U = \frac{1}{2}\int_0^{L_x}\int_0^{L_y}\left(\kappa_x \quad \kappa_y \quad \kappa_{xy}\right)\begin{matrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66}\end{matrix}\quad \begin{matrix}\kappa_x \\ \kappa_y \\ \kappa_{xy}\end{matrix}\quad dydx$$

(3)

When a plate undergoes free, undamped vibration the deflection of the plate is sinusoidal with respect to time t

$$w^o = \overline{w}^o \mathrm{Sin}\left(\omega t\right) = \overline{w}^o \mathrm{Sin}\left(2\pi ft\right) \qquad (4)$$

The relationship between curvature and the deflections

$$\begin{matrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{matrix} \quad = \quad \begin{matrix}\dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x}\end{matrix} \quad + Z \quad \begin{matrix}\dfrac{\partial^2 w_o}{\partial x^2} \\ \dfrac{\partial^2 w_o}{\partial y^2} \\ 2\dfrac{\partial^2 w_o}{\partial x \partial y}\end{matrix}$$

(5)

By using equation (5) we get

$$\overline{U} = \frac{1}{2}\int_0^{Lx}\int_0^{Ly}[\ D_{11}(\frac{\partial^2 w_o}{\partial x^2})^2 + D_{22}(\frac{\partial^2 w_o}{\partial y^2})^2 + D_{66}(\frac{2\partial^2 w_o}{\partial x\ \partial y})^2$$

$$+ 2(D_{12}\frac{\partial^2 w_o}{\partial x^2}\frac{\partial^2 w_o}{\partial y^2} + D_{16}\frac{\partial^2 w_o}{\partial x^2}\frac{2\partial^2 w_o}{\partial x \partial y}$$

$$+ D_{26}\frac{\partial^2 w_o}{\partial y^2}\frac{2\partial^2 w_o}{\partial x \partial y})]dydx \qquad (6)$$

Following whitney [6], we obtain the natural frequencies of this plate by the energy method. By introducing w_0 into the expression for U we obtain

$$U = \overline{U} \quad Sin^2(2\pi ft) \qquad (7)$$

The kinetic energy of the plate is

$$K = \frac{1}{2}\int_0^{L_x}\int_0^{L_y}\rho(\frac{dw^o}{dt})^2\, dydx. \qquad (8)$$

Substituting of the deflection into this equation yields

$$K = \frac{1}{2}(2\pi f)^2 \cos^2(2\pi ft)\int_0^{L_x}\int_0^{L_y}\rho\overline{w_o}^2 dydx \qquad (9)$$

According to the law of conservation of energy the change in strain energy from time t=0 to time t equals the change in kinetic energy during this time

$\left(U_t \quad U_{t=0}\right) = \left(K_t \quad K_{t=0}\right)$

Initially, at time t=0 the strain energy is zero equation of U but at time $t = \dfrac{1}{4f}$ the kinetic energy is zero, thus we have

$$U_{t=\frac{1}{4f}} = K_{t=0} \qquad (10)$$

Now using from equation (8) and (9) into equation (10)

$$\frac{1}{2}(2\pi f)^2\int_0^{L_x}\int_0^{L_y}\rho\overline{w}_o^{\,2} dydx = \overline{U} \qquad (11)$$

For the simply supported plate under consideration the geometrical boundary conditions require the deflections by zero along the edge

$$w^o = 0 \quad \text{at} \quad \begin{matrix} x = 0 & \text{and} & 0 \le y \le L_y \\ x = L_x & \text{and} & 0 \le y \le L_y \\ 0 \le x \le L_x & \text{and} & y = 0 \\ 0 \le x \le L_x & \text{and} & y = L_y \end{matrix}$$

(12)

The following deflection satisfies these geometrical boundary conditions

$$w^o = \overline{w}^o Sin(2\pi ft)$$

Where $\overline{w}^o$ is

$$\overline{w^o} = \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij} \sin\frac{i\pi x}{L_x}\sin\frac{i\pi y}{L_y}, \qquad (13)$$

Where I and J are the number of terms, chosen arbitrarily, for the summations and w_{ij} are constants. According to the Rayleigh principle the frequency of vibration of a conservative system has a minimum value in the neighborhood of the fundamental mode. We express this principle in the form

$$\frac{\partial f}{\partial w_{ij}} = 0, \qquad (14)$$

By substituting equation (13) into equation (14) the natural frequencies are calculated as follows

$$f_{ij} = \frac{1}{\pi}\sqrt{\frac{\lambda_{ij}}{\rho L_x L_y}} \quad (15)$$

Using Rayleigh's energy method [7] the natural frequencies of an orthotropic plate is given by following equation

$$f_{ij} = \sqrt{\frac{\pi^2}{4\rho}\left\{D_{11}\left(\frac{i}{L_x}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{i}{L_x}\right)^2\left(\frac{j}{L_y}\right)^2 + D_{22}\left(\frac{j}{L_y}\right)^4\right\}} \quad (16)$$

Natural frequencies of CFRP plates are measured by ANSYS and compared with results from Equation (15). During FEM formulation to calculate natural frequency following equation is used

$$[K]\{\phi_i\} = \omega_i^2[M]\{\phi_i\} \quad (17)$$

Where [K] is stiffness matrix, $\{\phi_i\}$ is mode shape vector of mode i, ω_i is the natural circular frequency, ω_i^2 is the eigen value and [M] is the mass matrix.

3. FINITE ELEMENT MODELING

The finite element program ANSYS is used to study the thermal effects on the dynamic properties of CFRP plates under simply supported and clamped boundary conditions. An eight node layered shell element (Shell 99) is used to model the layered plate. The Shell99 is an eight nodded linear layered structural shell element that can take maximum of 250 equal thickness layers. This element has six degree of freedom at each node; three translation x,y,z and three rotation about the x,y,z axes. The formulation of the elements includes first order shear deformation plate theory

4. RESULTS AND DISCUSSION

At first, the present FE model has been validated by comparing its results with those of Equation (16) for simply supported $[0, \pm45, 90]_s$ composite laminated plate. Then study involves a parameter study of the free vibration response, by using the finite element method, of a number of angle-ply and crossed ply clamped (symmetric and unsymmetric) laminated plates. The plates are subjected to uniform temperature change. Initially, no coefficient of thermal expansion is included, in order to observe effects of variation of material properties only with temperature change. Finally coefficient of thermal expansion is included to study the effect of residual stresses and initial plate imperfections. The layup investigated are $[0_2, 90_2]$, $[0, 90]_4$, $[\pm45_2]$, $[\pm45]_4$, $[0, 90]_{2s}$, $[\pm45]_{2s}$, $[0, \pm45, 90]_s$ which denoted as Plate 1,Plate 2,Plate 3,Plate 4, Plate 5, Plate 6, Plate 7 respectively. The material properties used in these laminated plates are typically values measured for XAS/914c [8]. The frequencies ω are nondimensionalized according to

$$\omega_d = \omega a^2 (\rho/E_2)^{0.5}/h$$

Where,

ω_d = Nondimensional frequency
a = 0.3 m
ρ = 1630 kg/ m3
E_2 = 8.5 GPa

Figure 2 shows the variation of Fundamental frequency with temperature for different laminated composite plates. The investigation shows that the largest reduction of natural frequency with increasing temperature is occurred for the single cross ply, $[0_2, 90_2]$ and angle ply $[\pm45_2]$, laminates.

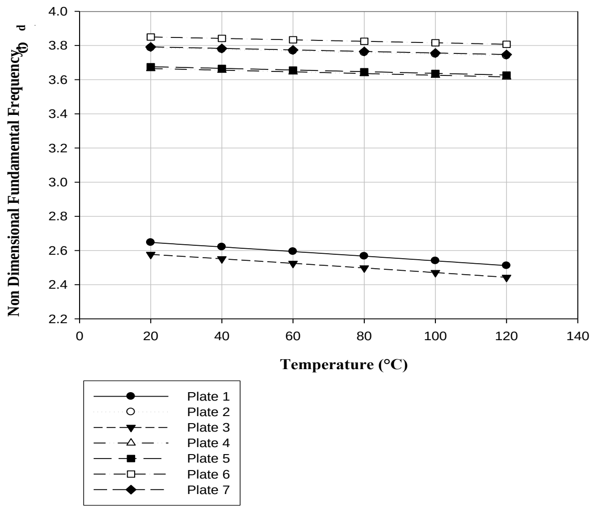

Fig 2. Variation of Nondimensional Fundamental Frequencies (**ω_d**) with Temperature(Without including coefficient of thermal Expansion).

It is appeared when the thermal expansion are not included, then increasing the number of cross or angled plies are significantly reduced the effect of temperature on the natural frequencies. Composite layup of $[0, 90]_{2s}$ and $[0, \pm45, 90]_s$ laminates has the higher value of fundamental frequency. Only slight variations of fundamental frequency with temperature are predicted for the multi-ply laminates.

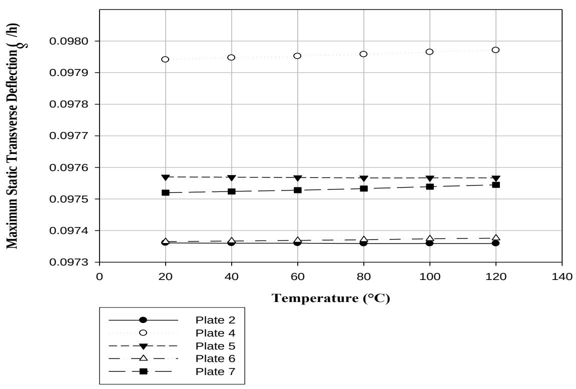

Fig 3. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup.

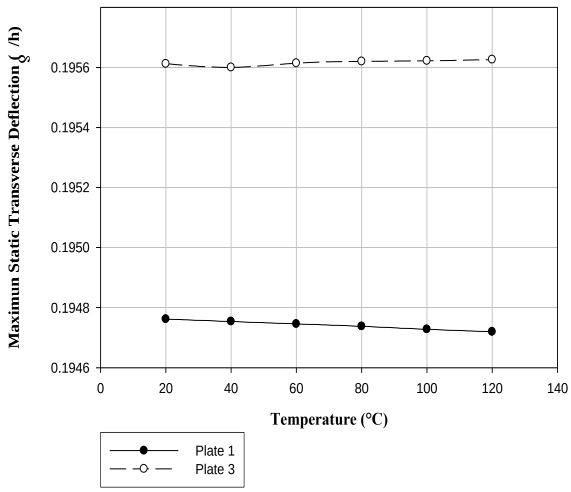

Fig 3. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup

Figures 3 & 4 show the maximum static transverse deflection with temperature. The value of maximum static transverse deflection is higher for Single cross ply, [0_2, 90_2] and angle ply [$\pm45_2$] and minimum for multiply [0, 90]$_4$, [0, 90]$_{2s}$ composites.

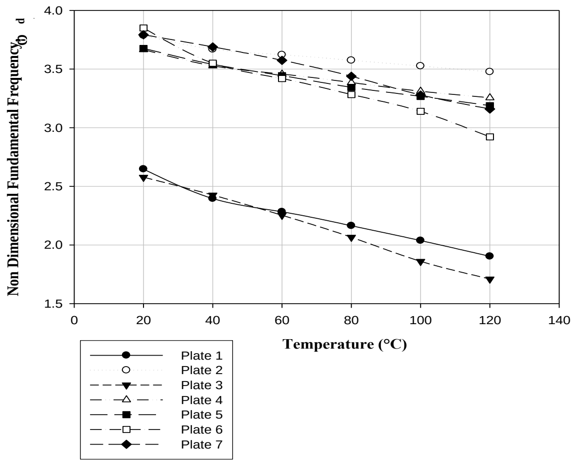

Fig 5. Variation of Nondimensional Fundamental Frequencies (**ω_d**) with Temperature (Including coefficient of thermal Expansion)

In general it appears that fundamental frequencies are decreased due to increasing the temperature without inclusion of thermal coefficient. These reductions of fundamental frequencies are more significant due to increasing temperature with inclusion of thermal coefficient.

The fundamental frequency is decreased drastically with temperature when coefficient of thermal expansion is included. Fundamental frequency of Single cross ply, [02, 902], angle ply [±452], and multiply symmetric laminates of [0, 90]2s has the severe effect with temperature as shown in the Figure 5. . For the composite laminate plate of [0, 90]4 layup has the less effect on natural frequency with temperature.

Figure 6 shows the variation of maximum static transverse deflection with temperature. As the reference temperature is set to 20°C so there is no prestress effect at 20°C. So the deflection at 20°C with inclusion of thermal coefficient is appeared as same as the deflection without inclusion of thermal coefficient. But at 40°C as the reference temperature is set to 20°C so the prestress effect has a significant effect on transverse deflection. Maximum static transverse deflection is reduced from 20°C to 40°C intensely.

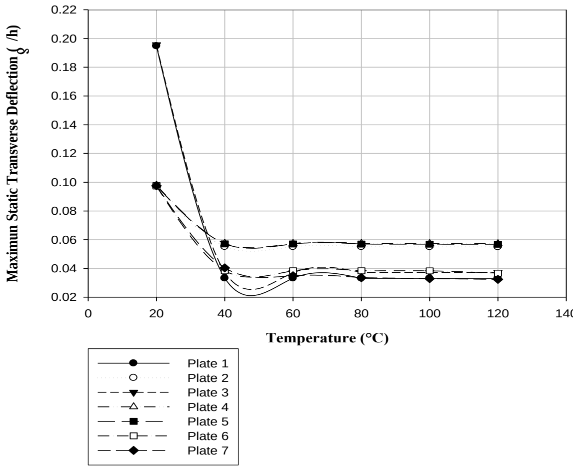

Fig 6. Variation of Maximum Static Transverse Deflection (d/h) with Temperature for composite plate of different layup.

The value of static transverse deflection is higher for composite plates of [±45]$_{2s}$ and minimum for [0_2, 90_2] and [0, ±45, 90]$_{2s}$ composite layup

A comparison between theoretical results with finite element results has been presented in this section. To validate the FEM results a comparison has been done between theoretical results and FEM results. Figure 7 shows the comparison between the theoretical results and finite element results of [0, ±45, 90]$_s$ Laminate. In both cases boundary condition is simply supported. Theoretical fundamental frequency is calculated by using equation (16). It can be seen that the finite element result are in good agreement with the theoretical values.

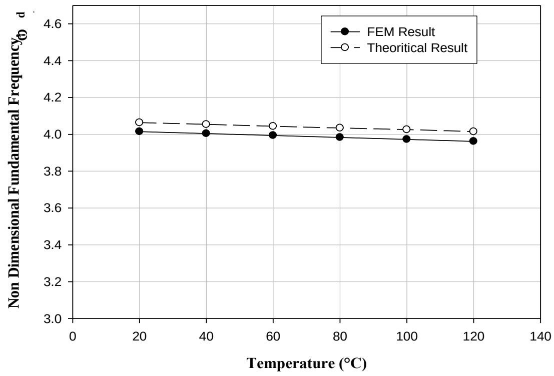

Fig 7. Comparison of Nondimensional Fundamental Frequencies (ω_d) for composite plate with theoretical Values.

Table 1: Variation of Fundamental Frequency with temperature (Without inclusion of Thermal coefficient).

Composite	20°C	40°C	60°C	80°C	100°C	120°C
$[0_2,90_2]$	2.647	2.620	2.594	2.56	2.535	2.511
$[0,90]_4$	3.786	3.772	3.767	3.75	3.741	3.739
$[\pm45]_2$	2.576	2.554	2.524	2.49	2.470	2.442
$[\pm45]_4$	3.664	3.655	3.645	3.63	3.625	3.615
$[\pm45]_{2s}$	3.675	3.666	3.656	3.64	3.631	3.625
$[0,90]_{2s}$	3.849	3.841	3.832	3.82	3.815	3.80
$[0,\pm45,90]_s$	3.791	3.782	3.773	3.76	3.75	3.747

Table 2: Variation of Fundamental Frequency with temperature (With inclusion of Thermal coefficient).

Composite	20°C	40°C	60°C	80°C	100°C	120°C
$[0_2,90_2]$	2.64	2.39	2.28	2.16	2.03	1.90
$[0,90]_4$	3.78	3.66	3.62	3.57	3.52	3.47
$[\pm45]_2$	2.57	2.42	2.2	2.06	1.85	1.70
$[\pm45]_4$	3.66	3.52	3.45	3.38	3.31	3.25
$[\pm45]_{2s}$	3.67	3.53	3.44	3.34	3.26	3.18
$[0,90]_{2s}$	3.84	3.54	3.41	3.28	3.13	2.92
$[0,\pm45,90]_s$	3.79	3.68	3.57	3.43	3.27	3.15

5. CONCLUSIONS

A parametric study was carried out with a number of cross-ply and angle ply square clamped laminates to observe the effect of uniform temperature and moisture content on the free vibration characteristics of such laminates.

The investigation shows that the largest reduction of natural frequency with increasing temperature (without prestress) is occurred for the single cross ply, $[0_2, 90_2]$ and angle ply $[\pm45_2]$, laminates. Only slight variations of fundamental frequency are predicted for the multi-ply laminates, $[0, 90]_{2s}$, [0, ±45, 90]s. For prestress effect, fundamental frequency is decreased 28.09% and 33.68% from 20°C to 120°C of composite plate Single cross ply, $[0_2, 90_2]$ and Angle ply $[\pm45_2]$ respectively. For the composite laminate plate of $[0, 90]_4$ layup has the less effect on fundamental frequency with temperature.

6. ACKNOWLEDGEMENT

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7. REFERENCES

1. B. L. Riley 1986 *22nd John Player Lecture, Institute of Mechanical Engineering proceedings* 20(50). AV-8B/GR MK5 airframe composite application.
2. Boley, B. A. and Weiner, J. H., Theory of Thermal Stresses. Wiley, New York (1960).
3. Fauconneau, G. and Marangoni, R. D., "Effect of a thermal gradient on the natural frequencies of a rectangular plate." *Int. J. Mech. Sci.* 12, 113 (1970).
4. Bailey, C. D., "Vibration of thermally stressed plates with various boundary conditions." *AIAA J.* 11, 14 (1973).
5. Jones, R., Mazumdar, J. and Cheung, Y. K., "Vibration and buckling of plates at elevated temperatures." *Int. J. Mech. Sci.* 16, 61 (1980).
6. J. M, Whitney, *structural Analysis of Laminated Anisotropic plates*, Lancester, Pennsylvania, 1987, p.166
7. L. Meirovitch, *Principles and Technique of Vibrations*, prentice-Hall, upeer saddle River, New Jerssey, 1997, pp. 518-522
8. S. C. P. Galea 1989 Ph. D. Thesis, ISVR, University of Southampton, U.K., *The effects of temperature on the acoustically induced strains and damage propagation in CFRP plates.*

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